GCSE Maths – Ratio, Proportion and Rates of Change

Direct and Inverse Proportion

Worksheet

WORKED SOLUTIONS

This worksheet will show you how to work out different types of direct and inverse proportion questions. Each section contains a worked example, a question with hints and then questions for you to work through on your own.

Section A

Worked Example

10 apples cost £2.40, how much will it cost to buy 9 apples?

Step 1: Find the cost of one apple

$$2.40 \div 10 = 0.24$$

$\div 10$ *10 apples : £2.40* $\div 10$
1 apple : £0.24

Step 2: Find the cost of 9 apples

$$0.24 \times 9 = 2.16$$

$\times 9$ *1 apple : £0.24* $\times 9$
9 apples : £2.16

9 apples cost £2.16

Guided Example

5 bananas cost £3.60, how much will it cost to buy 7 bananas?

Step 1: Find the cost of one banana

÷5 to find value of 1 banana

5 bananas = £3.60
1 banana = £3.60 ÷ 5
= £0.72

Step 2: Find the cost of 7 bananas

x7 to find value of 7 bananas

1 banana = £0.72
7 bananas = £0.72 × 7
= £5.04

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

1. 6 pens cost £2.16. Calculate the cost of 12 pens.

6 pens = £2.16
1 pen = £2.16 ÷ 6
= £0.36

12 pens = 1 pen × 12
= £0.36 × 12
= £4.32

[or, 12 is 6×2 so cost of 12 pens = £2.16 × 2 = £4.32]

2. 8 water bottles cost £20. Calculate the cost of 13 water bottles.

8 water bottles = £20
1 water bottle = £20 ÷ 8
= £2.50

13 water bottles = 1 water bottle × 13
= £2.50 × 13
= £32.50

3. Maya buys 7 nail polishes for £10.57. Calculate the cost of 15 nail polishes.

7 nail polishes = £10.57
1 nail polish = £10.57 ÷ 7
= £1.51

15 nail polishes = 15 × 1 nail polish
= 15 × £1.51
= £22.65

4. Raf bought 9 earrings for £9.81. Ayushi bought 7 earrings for £7.42. Who got the better value?

Raf:
9 earrings = £9.81
1 earring = £9.81 ÷ 9
= £1.09

Ayushi:
7 earrings = £7.42
1 earring = £7.42 ÷ 7
= £1.06

Ayushi paid £1.06 per earring and Raf paid £1.09. £1.06 < £1.09, so Ayushi got better value.

Section B

Worked Example

A is directly proportional to the square root of B. When $A = 16$, $B = 16$.
Find A when $B = 81$.

Step 1: Write an equation involving k

this symbol means 'directly proportional to'.

$$A \propto \sqrt{B}$$
$$A = k\sqrt{B}$$

Step 2: Substitute the known values into the equation

$$A = k\sqrt{B}$$
$$16 = k\sqrt{16}$$

Step 3: Solve for k

$$16 = k\sqrt{16}$$
$$16 = 4k$$
$$k = 4$$

Step 4: Express A in terms of B

$$A = 4\sqrt{B}$$

Step 5: Find the value for A

$$A = 4\sqrt{81}$$
$$A = 4 \times 9$$
$$A = 36$$

Guided Example

T is directly proportional to the square of U. When $T = 16$, $U = 2$.
Find U when $T = 64$.

Step 1: Write an equation involving k

$$T \propto u^2$$
$$T = ku^2$$

where k is a constant.

Step 2: Substitute the known values into the equation

when $T = 16$, $u = 2$:

$$16 = k(2^2) \leftarrow \text{apply BIDMAS}$$
$$T = k(u^2)$$

↳ indices before multiplication

Step 3: Solve for k

$$16 = k(4)$$
$$\div 4 \qquad 16 \div 4 = k \qquad \div 4$$
$$4 = k$$

Step 4: Express T in terms of U

$$T = ku^2$$
$$T = 4(u^2)$$

Step 5: Find the value for U

Rearrange for u:

$$T = 4(u^2)$$
$$\div 4 \qquad \frac{T}{4} = u^2 \qquad \div 4$$
$$\sqrt{\ } \qquad \sqrt{\frac{T}{4}} = u \qquad \sqrt{\ }$$

when $T = 64$:

$$u = \sqrt{\frac{64}{4}}$$
$$u = \sqrt{16}$$
$$u = 4$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

5. X is directly proportional to the square of Y. When $X = 50$, $Y = 5$.
   Find X when $Y = 3$.

$x \propto y^2$
$x = ky^2$
Substitute known values of $x=50$ and $y=5$ → $50 = k(5^2)$
$50 = k(25)$
$2 = k$

$x = ky^2$
$x = 2(y^2)$
when $y = 3$:
$x = 2(3^2)$
$x = 2(9)$
$x = 18$

6. C is directly proportional to the cube root of D. When $C = 32$, $D = 8$.
   Find D when $C = 16$.

$C \propto \sqrt[3]{D}$
$C = k \times \sqrt[3]{D}$
$32 = k \times \sqrt[3]{8}$
$32 = k \times 2$
$16 = k$

$C = 16 \times \sqrt[3]{D}$
when $C = 16$:
$16 = 16 \times \sqrt[3]{D}$ ÷16
$1 = \sqrt[3]{D}$ D^3 $(1)^3$
$1 = D$

7. P is directly proportional to Q. When $P = 14$, $Q = 5$.
   Find Q when $P = 6$.

$P \propto Q$
$P = kQ$
$14 = k \times 5$
$\frac{14}{5} = k$

$P = \frac{14}{5}Q$
when $P = 6$:
$6 = \frac{14}{5}Q$
$6 \div \frac{14}{5} = Q$
$\frac{15}{7} = Q$

8. Lauren is paid £225 for 25 hours of work. Use direct proportion to calculate how much she is paid for 30 hours of work.

let H = hours worked, P = pay.

$P \propto H$
$P = kH$
when $H = 25$, $P = 225$:
$225 = k \times 25$
$225 \div 25 = k$
$9 = k$

$P = kH$
$P = 9H$
when $H = 30$:
$P = 9 \times 30$
$P = 270$
∴ she is paid £270 for 30 hours of work.

Section C

Worked Example

The time taken (t) for customers to be served is inversely proportional to the square root of the number of waiters (w) working. It takes 10 min to be served when there are 4 waiters working. Find t in terms of w.

Step 1: Write an equation involving k.

$$t \propto \frac{1}{\sqrt{w}}$$

$$t = \frac{k}{\sqrt{w}}$$

Step 2: Substitute the known values into the equation

$$t = \frac{k}{\sqrt{w}}$$

$$10 = \frac{k}{\sqrt{4}}$$

Step 3: Solve for k.

$$10 = \frac{k}{\sqrt{4}}$$

$$10 = \frac{k}{2}$$

$$k = 20$$

Step 4: Express t in terms of w

$$t = \frac{20}{\sqrt{w}}$$

Guided Example

T is inversely proportional to the square of U. When $T = 7$, $U = 3$.
Find T when $U = \sqrt{21}$.

Step 1: Write an equation involving k.

$$T \propto \frac{1}{u^2}$$

$$T = k \times \frac{1}{u^2}$$

$$T = \frac{k}{u^2}$$

Step 2: Substitute the known values into the equation.

when $T = 7$, $u = 3$:

$$T = \frac{k}{u^2} \qquad 7 = \frac{k}{3^2}$$

$$7 = \frac{k}{9}$$

Step 3: Solve for k.

$$7 = \frac{k}{9}$$

×9 ×9

$$9 \times 7 = k$$

$$63 = k$$

Step 4: Express T in terms of U.

$k = 63$

$$T = \frac{k}{u^2}$$

$$T = \frac{63}{u^2}$$

Step 5: Find the value for T.

when $u = \sqrt{21}$:

$$T = \frac{63}{(\sqrt{21})^2}$$

$$T = \frac{63}{21}$$

$$T = 3$$

Now it's your turn!
If you get stuck, look back at the worked and guided examples.

9. X is inversely proportional to the square of Y. When $X = 2$, $Y = 5$.
   Find X when $Y = \sqrt{10}$.

$x \propto \frac{1}{y^2}$

$x = \frac{k}{y^2}$

$2 = \frac{k}{5^2}$

$2 = \frac{k}{25}$

$2 \times 25 = k$

$50 = k$

$x = \frac{50}{y^2}$

$x = \frac{50}{(\sqrt{10})^2}$

$x = \frac{50}{10}$

$x = 5$

10. C is inversely proportional to the cube root of D. When $C = 4$, $D = 8$.
    Find D when $C = 2$.

$C \propto \frac{1}{\sqrt[3]{D}}$

$C = \frac{k}{\sqrt[3]{D}}$

values from question $\rightarrow$ $4 = \frac{k}{\sqrt[3]{8}}$

$4 = \frac{k}{2}$

$k = 2 \times 4$

$k = 8$

$C = \frac{8}{\sqrt[3]{D}}$

when $C = 2$:

$2 = \frac{8}{\sqrt[3]{D}}$

$2 \times \sqrt[3]{D} = 8$

$\sqrt[3]{D} = \frac{8}{2}$

$\sqrt[3]{D} = 4$

$D = 4^3$

$D = 64$

11. P is inversely proportional to the Q. When $P = 34$, $Q = 9$. Find Q when $P = 5$

$P \propto \frac{1}{Q}$

$P = \frac{k}{Q}$

$34 = \frac{k}{9}$

$k = 9 \times 34$

$k = 306$

$P = \frac{306}{Q}$

when $P = 5$:

$5 = \frac{306}{Q}$

$5Q = 306$

$Q = \frac{306}{5} = 61.2$

12. The number of days (d) to complete a bedroom renovation is inversely to the square of the number of workers (w). It takes 25 days for 2 workers to complete it. Calculate to the nearest day, how long it would take 10 workers to complete the job.

$d \propto \frac{1}{w^2}$

$d = \frac{k}{w^2}$

when $d = 25$, $w = 2$:

$25 = \frac{k}{2^2}$

$25 = \frac{k}{4}$

$100 = k$

$d = \frac{100}{w^2}$

when $w = 10$:

$d = \frac{100}{10^2}$

$d = \frac{100}{100}$

$d = 1$

When there are 10 workers, it takes 1 day to complete the renovation.